G WHERE FUNCTIONS MEET

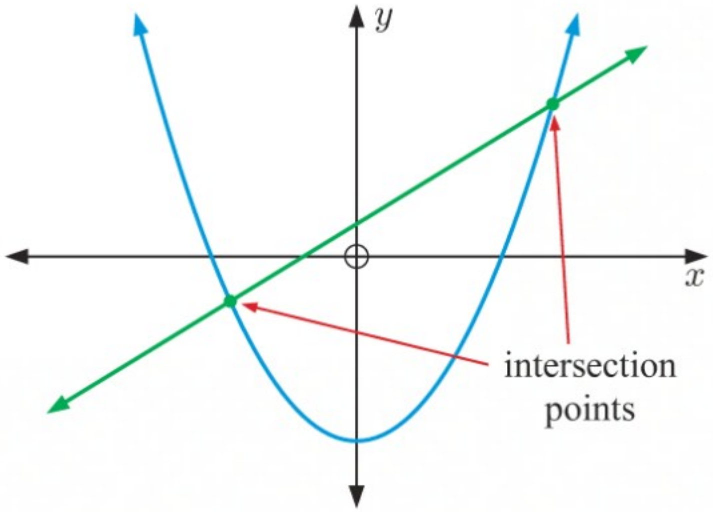
We often draw the graphs of two functions on the same set of axes, and are interested in finding where the functions meet.

CARTESIAN BAR
SO, DO YOU COME HERE OFTEN?
X²
X³

To find the points of intersection of the graphs of $y = f(x)$ and $y = g(x)$, we solve the equation $f(x) = g(x)$.

The solutions of this equation give us the x-coordinates of the intersection points.

The y-coordinates can then be found by substituting the x-coordinates into one of the functions.



Example 11

Self Tutor

Find the coordinates of the points of intersection of the graphs with equations $y = x^2 - x + 3$ and $y = 2x + 7$.

Example 11 **Self Tutor**

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The graphs meet when $x^2 - x + 3 = 2x + 7$

$$\therefore\ x^2 - 3x - 4 = 0$$

$$\therefore\ (x+1)(x-4) = 0$$

$$\therefore\ x = -1 \text{ or } 4$$

Substituting into $y = 2x + 7$: when $x = -1$, $y = 2(-1) + 7 = 5$

when $x = 4$, $y = 2(4) + 7 = 15$

$\therefore$ the graphs meet at $(-1,\ 5)$ and $(4,\ 15)$.

EXERCISE 13G

1 Find the coordinates of the point of intersection of the graphs with equations:

a $y = 5x - 1$ and $y = 2x + 5$

b $y = 12 - x$ and $y = 3x + 7$

c $y = \frac{1}{x}$ and $y = \frac{2}{x+5}$

d $y = x^2 - x + 3$ and $y = x^2 + 5x - 3$.

2 Find the coordinates of the point(s) of intersection of the graphs with equations:

a $y = x^2 + 2x - 1$ and $y = x + 5$

b $y = \frac{2}{x}$ and $y = x - 1$

c $y = 3x^2 + 4x - 1$ and $y = x^2 - 3x - 4$

d $y = \frac{1}{x}$ and $y = 5x - 4$.

3 Use a **graphing package** or a **graphics calculator** to find the coordinates, correct to 2 decimal places, of the points of intersection of the graphs with equations:

a $y = x^2 + 3x + 1$ and $y = 2x + 2$

b $y = x^2 - 5x + 2$ and $y = \frac{3}{x}$

c $y = -x^2 - 2x + 5$ and $y = x^2 + 7$

d $y = x^2 - 1$ and $y = x^3$.

GRAPHING PACKAGE

GRAPHICS CALCULATOR INSTRUCTIONS

1 **a** $(2,\ 9)$ **b** $(\frac{5}{4},\ \frac{43}{4})$ **c** $(5,\ \frac{1}{5})$ **d** $(1,\ 3)$

2 **a** at $(-3,\ 2)$ and $(2,\ 7)$ **b** at $(2,\ 1)$ and $(-1,\ -2)$
c at $(-\frac{1}{2},\ -\frac{9}{4})$ and $(-3,\ 14)$ **d** at $(1,\ 1)$ and $(-\frac{1}{5},\ -5)$

3 **a** at $(-1.62,\ -1.24)$ and $(0.62,\ 3.24)$ **b** at $(4.71,\ 0.64)$
c They do not intersect, $\therefore$ no solutions exist.
d at $(-0.75,\ -0.43)$